12 Days of Christmas

Gift Total Calculations

Sum of gifts = Day 1 gifts + Day 2 gifts + Day 3 gifts + . . . + Day 12 gifts

$= (1) + (2 + 1) + (3 + 2 + 1) + \cdots + (12 + 11 + 10 + \cdots + 1)$

So, on the n^th day, there are $\sum_{k=1}^{n} k$ gifts.

$$\sum_{n=1}^{4}(3n+2)$$

"Sigma notation" is a shortcut for expressing sums: it means the sum of everything in the brackets; in this example, starting with n=1 going up to n=4. So in this example, $(3 \times 1 + 2) + (3 \times 2 + 2) + (3 \times 3 + 2) + (3 \times 4 + 2)$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

On the n^th day
there are $\frac{n(n+1)}{2}$ gifts.

A useful fact to have up your sleeve:

$$\sum_{k=1}^{n}(k) = \frac{n(n+1)}{2}$$

We need to sum this over the 12 days:

$$\begin{aligned}
&\sum_{n=1}^{12} \frac{n(n+1)}{2} \\
&= \sum_{n=1}^{12}\left(\frac{n^2+n}{2}\right) \\
&= \sum_{n=1}^{12}\left(\frac{n^2}{2}+\frac{n}{2}\right) \\
&= \frac{1}{2}\sum_{n=1}^{12} n^2 + \frac{1}{2}\sum_{n=1}^{12} n \\
&= \frac{1}{2} \times \frac{(12)(12+1)(2 \times 12+1)}{6} + \frac{1}{2} \times \frac{12(12+1)}{2} \\
&= \frac{12 \times 13 \times 25}{12} + \frac{12 \times 13}{4} \\
&= 364
\end{aligned}$$

A less well known, and less frequently useful fact to have up your sleeve (we didn't; we had to figure it out ☺):

$$\sum_{n=1}^{N}(n^2) = \frac{N(N+1)(2N+1)}{6}$$

And so we can show that if someone were to receive all those gifts from the 12 Days of Christmas song, there would be a total of one gift for every day of the year EXCEPT Christmas Day!