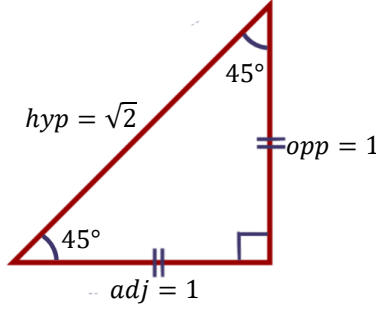
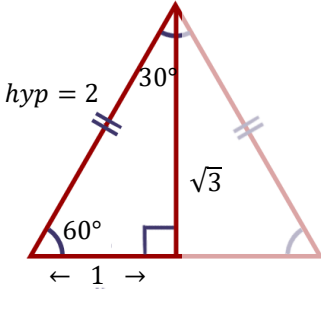


Working With Special Triangles

Tips and Tricks

There are two special triangles that give us exact values for the trig ratios for three different angles: 30° , 45° , and 60° . When we want to know the exact value of a trig ratio (as opposed to a portion of the never-ending decimal that our calculator gives us) we use these special triangles.

Creating the Special Triangles

		Remember SOH CAH TOA ! $\sin \theta = \frac{opp}{hyp} ; \cos \theta = \frac{adj}{hyp} ; \tan \theta = \frac{opp}{adj}$ hyp = hypotenuse = side opposite 90° opp = side opposite the angle we are working with adj = side adjacent to (beside) angle we are working with
How it Works ➤ The sum of the angles of a triangle is always 180° ➤ Two angles of an isosceles triangle are equal ➤ The right angle is 90° ➤ ∴ angle = (180 – 90)/2 = 45° ➤ Set the length of the two equal sides to be 1 ➤ Using Pythagorean Theorem: ○ 1² + 1² = hyp² ∴ hyp = √2	How it Works ➤ Draw an equilateral triangle. ➤ Mark all angles as 60° and all sides as equal. ➤ Draw a vertical line from the top vertex to the base ➤ Label the 90° angle between the base and the vertical ➤ Use one of the right triangles created. ➤ Label the length of one side as 2; this gives us the “half side” at the base as 1. ➤ Using Pythagorean Theorem: ○ 1² + opp² = 2² ∴ opp = √3	
$\sin 45^\circ = \frac{1}{\sqrt{2}} ; \cos 45^\circ = \frac{1}{\sqrt{2}} ; \tan 45^\circ = 1$	$\sin 30^\circ = \frac{1}{2} ; \cos 30^\circ = \frac{\sqrt{3}}{2} ; \tan 30^\circ = \frac{1}{\sqrt{3}}$	$\sin 60^\circ = \frac{\sqrt{3}}{2} ; \cos 60^\circ = \frac{1}{2} ; \tan 60^\circ = \sqrt{3}$

A Tip for Working with Special Triangles

Get into the habit of doing a quick sketch of the two special triangles on your work page when you start working. Use your sketch as a quick reference to confirm the trig ratios for the special angles and to confirm the angles if you are given exact trig ratios.

An Interesting Observation

There is an interesting pattern to the sine and cosine values for the angles 0° , 30° , 45° , 60° , and 90° . See the table to the right.

Note the consistency: before simplifying, all values have 2 as the denominator, all have a square root in the numerator;

Note the pattern: before simplifying, the numerator under the square root increases from 0 to 4 for sine and decreases from 4 to 0 for cosine.

Simplifying:

$$\frac{\sqrt{4}}{2} = \frac{2}{2} = 1; \quad \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{1}{\sqrt{2}}$$

Angle	Sine Value	Cosine Value
0°	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{4}}{2} = 1$
30°	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2}$
45°	$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$	$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$
90°	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{0}}{2} = 0$